Name: Solutions

Start Time:________________

End Time:________________

Date:________________

Math 260

Quiz 3 (30 min)

1. (1, 1, 1, 1, 2 points) If $A = \begin{bmatrix} 2 & 0 & -5 \\ -7 & 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 4 & 7 \\ -9 & 1 & -2 \end{bmatrix}$, find

a) $A + B$

$$= \begin{bmatrix} -2 & 4 & 2 \\ -16 & 4 & -1 \end{bmatrix}$$

b) $B - A$

$$= \begin{bmatrix} -6 & 4 & 12 \\ -2 & -2 & -3 \end{bmatrix}$$

c) $-2B$

$$= \begin{bmatrix} 8 & -8 & -14 \\ 18 & -2 & 4 \end{bmatrix}$$

d) $-A$

$$= \begin{bmatrix} -2 & 0 & 5 \\ 7 & -3 & -1 \end{bmatrix}$$

e) $4A - 3B$

$$= \underbrace{\begin{bmatrix} 8 & 0 & -20 \\ -28 & 12 & 4 \end{bmatrix}}_{4A} - \underbrace{\begin{bmatrix} -12 & 12 & 21 \\ -27 & 3 & -6 \end{bmatrix}}_{3B} = \boxed{\begin{bmatrix} 20 & -12 & -41 \\ -1 & 9 & 10 \end{bmatrix}}$$

2. (2 points) Prove: If $kA = 0$ (where k is a scalar and A is a matrix), then $k = 0$ or $A = 0$

proof: Let A be a matrix and k be a scalar such that $kA = 0$.

case 1: If $k = 0$, Then $k = 0$ or $A = 0$.

case 2: If $k \neq 0$, then multiply both sides of $kA = 0$ by $\frac{1}{k}$ (which exists since $k \neq 0$).

$$\frac{1}{k}(kA) = \frac{1}{k}0$$

$$\left(\frac{1}{k} \cdot k\right) A = 0$$

$$1A = 0$$

$A = 0$. So $k = 0$ or $A = 0$.

3. (2 points) Prove: If A and B are $m \times n$ matrices and k is a scalar, then $k(A + B) = kA + kB$

proof: Let A and B be $m \times n$ matrices and let K be a scalar.

Sizes

left

Since A and B both have size $m \times n$, $A+B$ has size $m \times n$ and so $K(A+B)$ also has size $m \times n$.

Same

right

Since A has size $m \times n$, KA also has size $m \times n$.

since B has size $m \times n$, KB also has size $m \times n$.

since both KA and KB have size $m \times n$, $KA+KB$ also has size $m \times n$.

Are corresponding entries equal?

Let $A = [a_{ij}]$ and $B = [b_{ij}]$.

left

$$K(A+B) = K([a_{ij}] + [b_{ij}]) = K[a_{ij} + b_{ij}]$$

$$= [K(a_{ij} + b_{ij})]$$

So the (i,j)-th entry of $K(A+B)$ is $K(a_{ij} + b_{ij})$.

right

$$KA + KB = K[a_{ij}] + K[b_{ij}] = [Ka_{ij}] + [Kb_{ij}]$$

$$= [Ka_{ij} + Kb_{ij}]$$

So the (i,j)-th entry of $KA+KB$ is $Ka_{ij} + Kb_{ij}$

Same

So $K(A+B) = KA + KB$.